

Second-class current effects from isospin breaking in $\tau \rightarrow \omega\pi\nu_\tau$

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Abstract

Second-class weak currents can in the standard model be induced by chiral-symmetry breaking. In the specific case of the decay $\tau \rightarrow \omega\pi\nu_\tau$, dominated by the first-class vector current with the ρ quantum numbers, such effects would manifest themselves by small axial vector (or, generally, non-vector) contributions to the decay rate. We present an attempt to estimate such effects, based on a vector and axial-vector dominance model of the relevant matrix elements supplemented by $\omega - \rho$ mixing. We also give an indication on the amplitude directly mediated by $b_1(1235) \rightarrow \omega\pi$, in principle also allowed in the standard model by isotopic spin violation.

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The weak currents coupled to W^\pm in semileptonic decays of hadrons composed of u and d quarks can be classified in terms of parity and G -parity as follows: first-class currents with $J^{PG} = 0^{--}, 1^{-+}, 1^{+-}$; second-class currents with $J^{PG} = 0^{+-}, 1^{++}$ [1, 2]. In the standard model with isospin (hence G -parity) conservation only first-class currents exist, and the transition $\tau \rightarrow \omega\pi\nu_\tau$ would proceed *via* a P-wave transition mediated by the 1^{-+} ‘ ρ -like’ vector current. In this situation, the contributions of an S- or a D-wave amplitude would unambiguously signal a $J^{PG} = 1^{++}$ second-class, non-standard axial current with the same quantum numbers as the meson $b_1(1235)$ [3]. Consequently, $\tau \rightarrow \omega\pi\nu_\tau$ has been considered as a sensitive test for the existence of second-class currents. With $\text{Br}(\tau \rightarrow \omega\pi\nu_\tau) = (1.99 \pm 0.08) \times 10^{-2}$ [4], the current experimental upper limit is for this decay: $\text{Br}(\text{second-class}) < 1.3 \times 10^{-4}$ at 90% CL [5].

It should be interesting to assess the size of the ‘second-class’ contributions to this decay generated in the standard model by isospin symmetry breaking, this would be useful to establishing the range in the genuine (non-standard) second-class currents coupling constants still allowed by the above mentioned upper limit for an eventual experimental discovery. The numerical estimates presented in the following will be purely phenomenological, in the sense that our modeling of isospin-breaking-generated second class currents will rely, to the largest possible extent within our knowledge, on input values for the needed coupling constants determined experimentally and quoted in [4].

Following Ref. [6], we separate the hadronic matrix element of the relevant $V - A$ weak current $J^\mu = \bar{\psi}_u \gamma^\mu (1 - \gamma_5) \psi_d$ into vector and axial vector parts as follows:

$$\langle \omega(k, \eta), \pi(p) | J^\mu | 0 \rangle = iV(s) \epsilon^{\mu\alpha\beta\gamma} \eta_\alpha k_\beta p_\gamma + A(s) \left(\eta^\mu - \frac{\eta \cdot p}{s} (k + p)^\mu \right). \quad (1)$$

Here: η^μ is the ω polarization vector, $\eta \cdot k = 0$; $s = q^2 = (k + p)^2$ is the $\omega\pi$ invariant mass squared; and $V(s)$ and $A(s)$ are the (dominant) vector and the (isospin violation suppressed) ‘second-class’ axial-vector form factors, respectively.¹

With $s_0 = (M_\omega + M_\pi)^2$ and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$, the partial decay

¹ Actually, the most general expansion would require two more form factors, one axial-vector and the other one scalar [6], but we here limit to the ones that according to our estimates are found to be numerically leading.

width can be written as:

$$\Gamma(\tau \rightarrow \omega\pi\nu_\tau) = \frac{G_F^2 |V_{ud}|^2}{1536\pi^3 M_\tau^3} \int_{s_0}^{M_\tau^2} \frac{ds}{s^2} \lambda^{1/2}(s, M_\omega^2, M_\pi^2) (M_\tau^2 - s)^2 (M_\tau^2 + 2s) \times \left[\lambda(s, M_\omega^2, M_\pi^2) |V(s)|^2 + \frac{\lambda(s, M_\omega^2, M_\pi^2) + 12sM_\omega^2}{2sM_\omega^2} |A(s)|^2 \right]. \quad (2)$$

The “forward-backward” asymmetry, which essentially counts the difference between numbers of events with positive and negative $\cos\theta$, with θ the $\pi - \tau$ angle in the $\omega\pi$ rest frame, is determined by the interference:

$$A_{\text{FB}} = \frac{1}{\Gamma(\tau \rightarrow \omega\pi\nu_\tau)} \frac{G_F^2 |V_{ud}|^2}{256\pi^3 M_\tau^3} \int_{s_0}^{M_\tau^2} \frac{ds}{s} \lambda(s, M_\omega^2, M_\pi^2) (M_\tau^2 - s)^2 \text{Re}[A(s)V^*(s)]. \quad (3)$$

In order to predict the observables (2) and (3), explicit expressions for the form factors $V(s)$ and $A(s)$ are needed.

Theoretical parametrizations for the dominant, first-class, form factor $V(s)$ mostly rely on vector meson exchange, see, for example, Refs. [6, 7]. We refer to the experimental resonance analysis of $\tau \rightarrow \omega\pi\nu_\tau$ of Ref. [8], and assume the simplified unsubtracted linear combination of $\rho \equiv \rho(770)$ and $\rho' \equiv \rho(1450)$ polar forms, see also Ref. [9]:

$$V(s) = \frac{\sqrt{2}F_\rho g_{\omega\rho\pi}}{M_\rho^2} \frac{1}{1 + \beta_\rho} \left[\frac{M_\rho^2}{M_\rho^2 - s} + \beta_\rho \frac{M_{\rho'}^2}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \right]. \quad (4)$$

In Eq. (4): $F_\rho \cong M_\rho^2/6$ is the $\rho \rightarrow e^+e^-$ coupling; for the $(\omega\rho\pi)$ coupling we take $g_{\omega\rho\pi} = 16.1 \text{ GeV}^{-1}$ [8]; and we choose the value of the constant $\beta_\rho \simeq -0.12$ in order to reproduce, from Eq. (2), the measured branching ratio of about 2%. Moreover, the s -dependent ρ' width is defined as [10, 11]:

$$\Gamma(s) = \theta(s - s_0) \frac{M_{\rho'}}{\sqrt{s}} \left(\frac{k(s)}{k_{\rho'}} \right)^3 \Gamma_{\rho'}, \quad (5)$$

where k denotes the momentum in the $\omega\pi$ c.m. frame. In a sense, Eq. (4) resembles the modification of the ρ propagator introduced in Ref. [12]. In Eq. (4), the width $\Gamma_\rho(s)$ has been omitted, since the ρ -pole is below the threshold s_0 , but this will have little impact on the numerical results. Indeed, considering also different, alternative, parametrizations of the s -dependent resonance widths, and the eventual inclusion of the width in the ρ pole, the values of β_ρ needed to reproduce the 2% branching ratio will ultimately range between -0.12 and -0.15 . It might be curious to notice that similar values of β_ρ have been calculated in

vector-dominance applications to second-class currents in τ semileptonic decays to $\eta\pi$ and $\eta'\pi$ [13].²

We model the contribution of the second-class axial current to $\tau \rightarrow \omega\pi\nu_\tau$ by the transition of τ to the axial-vector meson $a_1(1260)$, $\tau \rightarrow a_1\nu_\tau$, followed by $a_1 \rightarrow \rho\pi \rightarrow \omega\pi$ *via* the isospin violating $\rho - \omega$ mixing. Thus, defining the $a_1 \rightarrow \rho\pi$ transition matrix element as

$$T(a_1(q, \eta) \rightarrow \rho(k, \lambda) + \pi(p)) = (M_a^2 - M_\rho^2) (\eta \cdot \lambda) f_{a\rho\pi} + 2 (q \cdot \lambda) (k \cdot \eta) g_{a\rho\pi}, \quad (6)$$

where η and λ denote the a_1 and ρ polarization vectors, respectively, we would get for the axial form factor $A(s)$ the polar expression

$$A(s)|_{a_1} = \epsilon_{\omega\rho} f_a f_{a\rho\pi} \frac{M_a^2 - M_\rho^2}{M_a^2 - s - iM_a\Gamma_a(s)}. \quad (7)$$

In Eq. (7): $\epsilon_{\omega\rho}$ is the $\omega - \rho$ mixing parameter, and we simply assume $|\epsilon_{\omega\rho}| = 3 \times 10^{-2}$ from the branching ratio of $\omega \rightarrow 2\pi$ - this also averages, in some cases underestimates, determinations from the timelike pion form factor - see, e.g., [15]; for the constant f_a defined by $\langle 0 | \bar{\psi}_u \gamma_\mu \gamma_5 \psi_d | a_1(q, \eta) \rangle = f_a \eta_\mu$, we take $f_a \simeq 0.2 \text{ GeV}^2$, assuming the $\text{Br}(\tau \rightarrow 3\pi\nu_\tau) \simeq 10\%$ [4] to be saturated by the a_1 exchange; finally, the values of the constants $f_{a\rho\pi}$ and $g_{a\rho\pi}$ can be estimated from the $a_1 \rightarrow \rho\pi$ width.

In this regard, the a_1 width is rather badly known experimentally, Γ_{a_1} ranges from 250 to 600 MeV, while the situation is better for the D-wave/S-wave amplitude ratio in the transition $a_1 \rightarrow \rho\pi$, $D/S = -0.062 \pm 0.022$ [4]. From this ratio, using relations derived in Ref. [16], varying Γ_{a_1} in the range mentioned above and assuming $\text{Br}(a_1 \rightarrow \rho\pi)$ between 60% and 100%, we find the values $f_{a\rho\pi} \simeq 3.3 - 5.9$. For the coupling constant $g_{a\rho\pi}$ in Eq. (6), that would enter into the second axial form factor previously alluded to and found numerically suppressed, we would get $g_{a\rho\pi} \simeq 0.2 f_{a\rho\pi}$.

Using Eqs. (2) and (3) with the parametrizations (4) and (7) and the input parameters varied in the ranges indicated above, we finally obtain the following estimates for the isospin breaking second-class contributions:

$$\text{Br}(\tau \rightarrow \omega\pi\nu_\tau)|_{a_1} \simeq (1.6 - 2.1) \times 10^{-5}; \quad |A_{\text{FB}}| \simeq (2.4 - 4.8) \times 10^{-3}. \quad (8)$$

² For simplicity we do not include a non-resonant part of $V(s)$, that for soft pions can be evaluated in chiral perturbation theory [14].

As one can see, the uncertainty is rather large, and is mainly due to the extended range where the a_1 parameters can vary. However, the upper values in Eq. (8) are the most important ones for our purposes, in that they represent estimated limits for eventually observed second-class effects in $\tau \rightarrow \omega \pi \nu_\tau$ to be unambiguously considered as genuine, non-standard, signals rather than manifestations of symmetry breaking in the standard model.

An additional "second-class" axial-vector contribution from isotopic spin violation can be represented by the $b_1(1235)$ exchange, which we wish to parametrize analogously to Eq. (7). To this purpose, we recall that gluon corrections to the "bare" $\bar{u}dW$ vertex may generate a pseudotensor, divergenceless, coupling proportional to $\Delta m = m_d - m_u$, of the form [17, 18]

$$\bar{A}_\mu^{II}(x) = \bar{g}_T \partial^\nu \bar{\psi}_u(x) \sigma_{\mu\nu} \gamma_5 \psi_d(x) \equiv \bar{g}_T A_\mu^{II}; \quad \bar{g}_T = -\frac{4\alpha_s}{3\pi m} \frac{\Delta m}{2m}, \quad (9)$$

where m is the average quark mass. If in (9) one literally used current quark masses of the MeV order, the size of \bar{g}_T would be very large, of order 5 or more in GeV^{-1} units. However, this would be unjustified, because Eq. (9) strictly refers to free quarks. As discussed in Refs. [17–19], one expects that for confined quarks the loop integration over the gluon frequencies needed to derive this equation cannot run up to infinity, but must be cut-off at a scale appropriate to the hadronic scale. This will decrease the size of \bar{g}_T appreciably, in particular down to an order of magnitude compatible with phenomenological limits on second-class currents from nuclear β -decay, see, as an example, Refs. [20–22]. Accordingly, as a criterion to account for confinement effects in Eq. (9), we choose to input there the constituent quark masses $M_u \simeq M_d = 350 \text{ MeV}$, $\Delta M = 2 \text{ MeV}$, and $\alpha_s = 0.5$. This gives the indicative estimate $\bar{g}_T = -1.7 \times 10^{-3} \text{ GeV}^{-1}$.

We now need the pseudotensor constant $\langle 0 | \bar{\psi}_u \sigma_{\mu\nu} \gamma_5 \psi_d | b_1(q, \eta) \rangle = i f_b (\eta_\mu q_\nu - \eta_\nu q_\mu)$, for which we assume the quark-model value $f_b = \sqrt{2} f_a / M_b$ [23]. After contraction with q^ν as required by the expression (9), with $q^2 = M_b^2$ and $\eta \cdot q = 0$, we obtain for the second-class axial current matrix element: $\langle 0 | \bar{A}_\mu^{II} | b_1(q, \eta) \rangle = \bar{g}_T f_a \sqrt{2} M_b \eta_\mu$. With the $b_1 \rightarrow \omega \pi$ matrix element defined similar to Eq. (6),

$$T(b_1(q, \eta) \rightarrow \omega(k, \lambda) + \pi(p)) = (M_b^2 - M_\omega^2) (\eta \cdot \lambda) f_{b\rho\pi} + 2 (q \cdot \lambda) (k \cdot \eta) g_{b\rho\pi}, \quad (10)$$

we finally arrive at the following parametrization for the "direct" b_1 contribution to the form factor $A(s)$:

$$A(s)|_{b_1} = \bar{g}_T M_b \sqrt{2} f_a f_{b\omega\pi} \frac{M_b^2 - M_\omega^2}{M_b^2 - s - i M_b \Gamma_b(s)}. \quad (11)$$

With $\Gamma_{b_1} = 142 \text{ MeV}$, dominated by the b_1 -decay into $\omega\pi$, and the D/S amplitude ratio 0.277 [4], by a procedure similar to the case of the a_1 we obtain the value $f_{b\omega\pi} \simeq 5.0 \text{ GeV}^{-1}$ and in this way we complete the list of inputs needed in to numerically exploit Eq. (11). Finally, we represent the axial form factor $A(s)$ by the combination of a_1 and b_1 poles:

$$A(s) = A(s)|_{a_1} + A(s)|_{b_1}. \quad (12)$$

One can notice that, according to the above numerical estimates, the factor multiplying the b_1 Breit-Wigner form in (11) is suppressed with respect to the analogous factor multiplying the a_1 pole in (7) by about 10^{-1} . Possibly, this might be an overestimate of the b_1 contribution, did we choose for the mass scale in Eq. (9) the hadron mass, for example m_{b_1} , instead of the constituent quark masses, a smaller value of \bar{g}_T would have followed. Using Eq. (12), and the input values obtained above, we would find for the isospin-breaking induced second-class effects:

$$\text{Br}(\tau \rightarrow \omega\pi\nu_\tau)|_{A(s)} \simeq (2.3 - 2.8) \times 10^{-5}, \quad |A_{\text{FB}}| \simeq (2.6 - 5.3) \times 10^{-3}, \quad (13)$$

to be compared to the current upper limit on the second-class branching ratio of the order of 10^{-4} mentioned at the beginning. The numbers in (13) fall well-below that limit, and indicate that there still is ample room for an eventual discovery of second-class currents in the decay $\tau \rightarrow \omega\pi\nu_\tau$, before standard model, isospin breaking, effects are met. As regards the current situation with the pseudotensor genuine second-class current A_μ^{II} defined in (9), in β -decay it contributes as: $\langle p|A_\mu^{II}|n\rangle = g_T^{(\beta)} \bar{u}(p) i\sigma_{\mu\nu} \gamma_5 q^\nu u(n)$. Barring cancellations with other, non-pseudotensor, forms of second-class currents, the limits from different observables could be summarized by $|g_T^{(\beta)}| \leq (2 - 5) \times 10^{-1} \text{ GeV}^{-1}$ (see [22] and references therein). The nucleon matrix element of A_μ^{II} is suppressed by the smallness of the four-momentum q in β -decay. In $\tau \rightarrow \omega\pi\nu_\tau$ the momentum q is not small, this might enhance the sensitivity of this decay to A_μ^{II} . In fact, defining for a comparison a scale $g_T^{(\omega\pi)}$ analogous to $g_T^{(\beta)}$ and with same dimensions, and introducing it in Eq. (11) in place of \bar{g}_T , the current upper limit on the axial-vector branching ratio quoted at the beginning can exclude genuine second-class currents at the level $|g_T^{(\omega\pi)}| \leq 2 \times 10^{-2} \text{ GeV}^{-1}$.

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